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OPTIMAL DESIGN OF COMPLEX FLEXIBLE ROTOR-SUPPORT SYSTEMS USING MINIMUM STRAIN ENERGY UNDER MULTI-CONSTRAINT CONDITIONS

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The optimal design of complex flexible rotor-support systems is studied in this paper. Optimization using system strain energy is shown to be a convenient way to handle such systems. Multiple constraints such as the damped critical speeds, limitations on transmitted forces and the amplitudes of the deflection of shafts and disks, and stability considerations, are used to meet the engineering requirements. The support stiffnesses and clearances of squeeze film dampers (SFDs) are used as design variables. The transfer matrix-component mode synthesis method (TMCMS) is employed in the system dynamic analysis. A method of calculating damped critical speeds is also developed by using system strain energy criterion. The optimum results can be easily applied to preliminary engineering design as well as in modifications to existing machines.

1. INTRODUCTION

In the design of modern rotating machinery such as aircraft engines, gas turbines and compressors, there is an increasing requirement for high-speed, light-weight and high-performance. These considerations usually lead to the use of more flexible and more complex rotor systems. The rotors may be in the form of multi-level or branch structures having several disks, bearings and slender shafts. The trend towards greater flexibility results in critical speeds in or near the operational speed, which may cause severe vibration problems. The increasing complexity of the system makes both the system simulation and the design much more complicated due to the large number of parameters under consideration. Among these quantities, critical speeds, disk unbalance, the deflection of shafts or branches and the transmitted loads of bearings are the most important ones to be taken into account in the design process.

Significant work on the optimal design of rotor systems has been carried out by many researchers. Earlier work included approaches based on parameter sensitivity studies [1–3], giving guidance to practitioners on design improvements. Numerical optimal design

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methods have also been developed by other researchers to provide designers with more automated design tools. Generally speaking, the most commonly used objective functions are the following: (1) minimum weight [4–6]; (2) optimal arrangement of critical speeds [7, 8]; (3) minimum whirl amplitude of disks or deflection of shafts [9]; (4) minimum loads transmitted by bearings to the supports [10–12]. Two kinds of optimization variables were widely used in the previous studies. One is the geometry of the rotors, such as shaft diameters, disk sizes and the positions of bearings and disks. The other is the system support parameters, such as the stiffness and damping of bearings and supports and the oil viscosity. The use of system dimensions as variables is typical in minimum weight design. It may however prove cumbersome to use in practice, since more often than not, the dimensions of the rotor system and the position of bearings and disks are constrained by other considerations such as the overall engine structure and performance, and structural strength criteria, rather than dynamic performance. In this context, optimization of the support parameters offers an interesting alternative using the bearing and damper dimensions as variables, following references [9, 10, 12]. This in fact has proved to be a useful method for attaining optimal oil bearing and squeeze film damper (SFD) designs.

The previous work has contributed a lot to the understanding of the dynamic behavior of rotor systems and provided very useful tools for engineering design practice. However, most of the work reported in the literature has been related to relatively simple one-lever rotor systems. The work reported by Huang *et al*. [8, 11] on the optimal design of dual-spool rotor systems was one of the first attempts to address complex rotor system optimization.

Faced with a complex rotor system, a typical problem the design engineers often encounter is how to satisfy simultaneously quite a large number of requirements whilst constrained to include certain parameters. The use of local parameters as optimum objectives seems to be less realistic since it may be difficult to meet all the requirements and the results may even be in conflict with one another. One way to tackle this problem is the multi-objective optimization approach proposed by Shiau and Chang [13], and Miao and Huang [11]. In this case, the difficulty revolves around how to set up the relationship between different objectives. With increasing complexity, the task becomes even more challenging.

In this paper, an attempt is made to find a solution to such problems. A ‘‘global’’ quantity known as the minimum system strain energy is used as the optimum objective. The concept of minimum strain energy has already been used by Conry *et al*. [14] for the optimal unbalance distribution in flexible rotors. The idea is extended in the present study to achieve the optimal design of the whole system. It will be shown that the minimum strain energy offers a well-balanced option for all the aforementioned design requirements. Furthermore, it is a quadratic function leading to a unique minimum. In addition, by searching for the strain energy maxima over a given speed range, a method of calculating damped critical speeds is also developed. Also, multiple engineering constraints such as damped critical speeds, stability, limits on transmitted forces, amplitudes of disks and deflection of shafts are taken into account in the optimization process, thus meeting real engineering needs. As design variables, both the geometrical parameters of the structure and support parameters such as stiffness and damping coefficients may be used. The present study concentrates on the latter. These support parameter variables are shown to be very effective in modifying the system rotordynamic performance in terms of both critical speeds and unbalance responses. The transfer matrix-component mode synthesis method (TMCMS) developed by Huang [15, 16] was employed for the system dynamic analysis. The developed program can be used both at the preliminary optimal design stage and for modifications of the existing rotor system to improve dynamic performance.

2. SYSTEM DYNAMIC ANALYSIS

2.1. CALCULATION OF ROTORDYNAMIC UNBALANCE RESPONSES

Rotordynamic optimal design requires a suitable simulation method to calculate unbalance responses. In the present case, the method should be capable of handling complex rotor systems with reasonable efficiency.

Two simulation methods that have been widely used are the transfer matrix and finite element methods. The transfer matrix method is very effective for simple train structures. It is however difficult to use in the present case, involving complex components as described previously. The finite element method is more powerful, at the price of being more demanding as regards to computational capacity. Since complex systems usually have a large number of degrees of freedom, with iteration necessary in the optimization process, a more efficient method is needed, which can reduce the degrees of freedom for the calculation model. In the present paper, the so-called transfer matrix-component mode synthesis method (TMCMS) developed by Huang [16] is used for unbalance response calculations. This method uses the transfer matrix approach to compute the component modes of train-like subsystem while using the component mode synthesis method to reduce the degree of freedom of the whole system, thus retaining the advantages of both methods. The principle is briefly illustrated below.

The equation of motion for unbalanced rotor systems can be written as

$$[M]_{2n\times 2n}\{\ddot{p}\} \mp i\omega[C_g]_{2n\times 2n}\{\dot{p}\} + [C]_{2n\times 2n}\{\dot{p}\} + [K]_{2n\times 2n}\{p\} = \{M_e\}\omega^2, \tag{1}$$

where n is the total number of system lumped mass and inertial nodes.

TMCMS divides a complex system into several subsystems at the coupling nodes, as illustrated in Figure 1. Each subsystem is contrained at the boundary nodes, such as coupling nodes and nodes where bearings, elastic supports and dampers are used. System parameters are separated into boundary parameters related to each boundary point and inner parameters describing the inner nodes of the system. Equation (1) can then be expressed as

$$\begin{bmatrix} M_i & 0 \\ 0 & M_b \end{bmatrix}\begin{Bmatrix} \ddot{p}_i \\ \ddot{p}_b \end{Bmatrix} \mp i\begin{bmatrix} \omega_i C_{gi} & 0 \\ 0 & \omega_b C_{gb} \end{bmatrix}\begin{Bmatrix} \dot{p}_i \\ \dot{p}_b \end{Bmatrix} + \begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix}\begin{Bmatrix} p_i \\ p_b \end{Bmatrix}$$

$$+ \begin{bmatrix} C_{ii} & C_{ib} \\ C_{bi} & C_{bb} \end{bmatrix}\begin{Bmatrix} \dot{p}_i \\ \dot{p}_b \end{Bmatrix} = \{M_e\}\omega^2, \tag{2}$$

where subscripts i and b stand for the inner and boundary parameter matrix, respectively.

The motion of the whole system can be composed of a component mode shape assembly ϕ, consisting of a few low constrained undamped modes, the static deflection curves δ, and the system modal co-ordinates $\{q\}$. The so called static deflection curve δ corresponds to the deformation of the system when a unit displacement is imposed in turn at each boundary node. They are

$$\phi = [\phi_1, \phi_2, \ldots, \phi_m], \qquad \delta = [\delta_1, \delta_2, \ldots, \delta_k], \qquad \{q\} = \{q_1, q_2, \ldots, q_{m+k}\}^t.$$

Here, m is the number of constrained component modes, k is the number of boundary nodes. When they are separated into inner and boundary parameters, one has

$$\begin{Bmatrix} p_i \\ p_b \end{Bmatrix} = \begin{bmatrix} \phi_i & \delta_i \\ 0 & I_b \end{bmatrix}\{q\}. \tag{3}$$

The equation of motion for free vibration of the constrained undamped subsystem j is

$$[M_j]\{\ddot{p}_j\} \mp i\omega_j[C_{gj}]\{\dot{p}_j\} + [K_j]\{p_j\} = 0. \tag{4}$$

The eigensolution will take the form of

$$p_j = \phi_j \, e^{i\Omega_{cj}t}. \tag{5}$$

ϕ_j can be obtained through the constrained undamped subsystem eigensolution using an improved transfer matrix method which has been well explained by Huang [15, 16].

Let $y_i = 1$, $(i = 1, 2, \ldots, k)$, in turn at each time; the solution of ϕ_j under non-rotating condition gives the static deflection curves δ_j.

Taking differentials of equation (3) and substituting into equation (2), using the boundary conditions of stiffness and damping at the boundary nodes, one can obtain the system equation of motion in terms of the modal co-ordinates:

$$[\bar{M}]\ddot{q} + i[\bar{C}_g]\dot{q} + [\bar{K}]q + [\bar{C}]\dot{q} = [\Phi]^t\{M_e\}\omega^2, \tag{6}$$

where different terms are defined as follows (see Huang [15, 16]):

$$[\bar{M}]_{(m+k)\times(m+k)} = \begin{bmatrix} \phi^t M_i \phi & \phi^t M_i \delta \\ \delta^t M_i \phi & \delta^t M_i \delta + M_b \end{bmatrix},$$

$$[\bar{C}_g]_{(m+k)\times(m+k)} = \begin{bmatrix} \phi^t \omega C_{gi} \phi & \phi^t \omega C_{gi} \delta \\ \delta^t \omega C_{gi} \phi & \delta^t \omega C_{gi} \delta + \omega C_{gi} \end{bmatrix},$$

$$[\bar{K}]_{(m+k)\times(m+k)} = \begin{bmatrix} \phi^t(\Omega_c^2 M_i \mp \Omega_c \omega C_{gi})\phi & 0 \\ 0 & K_s + K_b \end{bmatrix},$$

$$[\bar{C}]_{(m+k)\times(m+k)} = \begin{bmatrix} 0 & 0 \\ 0 & C_b \end{bmatrix}, \qquad [\Phi]^t = \begin{bmatrix} \phi & \delta \\ 0 & I \end{bmatrix}^t.$$

Compared to equation (2), equation (6) has much smaller size since only a few important lower modes of the components are used. The accuracy of the method depends naturally on a good estimation and selection of the modes. Solutions of equation (6) give the modal

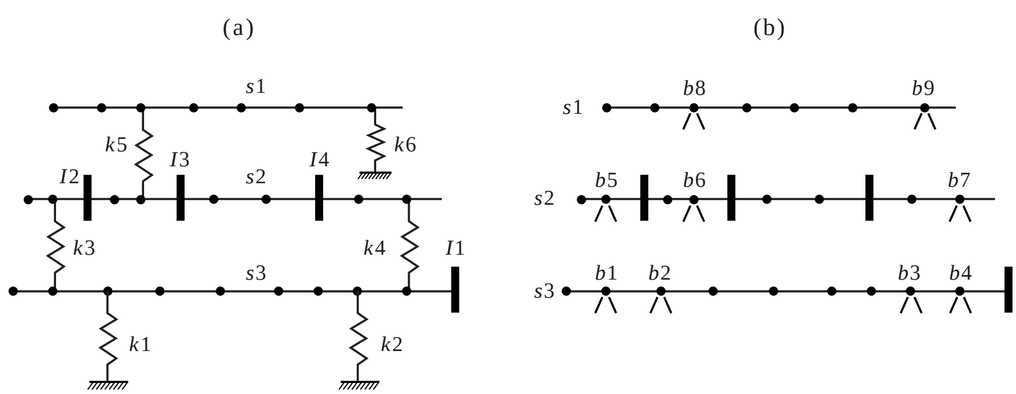

Figure 1. (a) Schematic of a typical complex rotor system, (b) substructures constrained on boundary nodes; b, boundary nodes; k, stiffness; I, inertia; s, subsystems; and ●, lumped mass nodes.

co-ordinate $\{q\}_{m+k}$ which allows one to use equation (3) to obtain the unbalance response in terms of the generalized displacements $\{p\}_{2n}$.

2.2. STRAIN ENERGY CALCULATION

Strain energy is an important quantity which reflects the system rotordynamic performance and strength states. It is defined as

$$U = \frac{1}{2}\int_v \sigma\epsilon \, \mathrm{d}v. \tag{7}$$

U can be divided into two parts: the energy of volume change U_v and energy of distortion U_d:

$$U = U_v + U_d. \tag{8}$$

According to the failure theories based on the energy of distortion [17], only the energy of distortion is responsible for failure due to inelastic action. In slender flexible structures, such as flexible rotors, the volume change U_v is small. As a result, the system strain energy has a direct effect on the strength of such structures.

The strain energy of the rotor-support system is a quadratic function expressed in terms of the system stiffness matrix $[K]$ and the generalized displacements $\{p\}$:

$$U = \tfrac{1}{2}\{p\}^*[K]\{p\}. \tag{9}$$

The total strain energy of the system is comprised of the sum of the strain energy of the rotor and that of the supports:

$$U = \sum_{i=1}^{n} U_{ri} + \sum_{j=1}^{k} U_{rj}, \tag{10}$$

where n is the number of component rotors and k is the number of supports. The strain energy of the supports is defined by

$$U_s = \tfrac{1}{2}k_e e^2, \tag{11}$$

where k_e is the support stiffness and e is the bearing eccentricity, two quantities affecting the forces transmitted by the bearings.

The size of the stiffness matrix $[K]_{2n\times 2n}$ and vector $\{p\}_{2n}$ is usually very large. Consequently, a direct calculation of strain energy using equation (11) is time-consuming. Due to the fact that the system strain energy is a scalar quantity independent of the co-ordinates, we can calculate it using modal co-ordinates:

$$U = \tfrac{1}{2}\{q\}^*[\bar{K}]\{q\}. \tag{12}$$

The size of $[\bar{K}]_{m+k}$ is much smaller than that of $[K]_{2n\times 2n}$, resulting in a significant reduction in both storage requirement and computing time.

2.3. CALCULATIONS OF DAMPED CRITICAL SPEEDS AND SYSTEM STRAIN ENERGY MAXIMA

Calculations of damped critical speeds in flexible rotor systems instead of undamped eigenvalues is important in the design process for several reasons. First, damping can greatly affect the dynamic behavior of the system: the real critical speeds may therefore be noticeably different from undamped eigenvalues. Moreover, strong damping can also make some rigid rotor modes disappear completely. Second, a knowledge of the exact

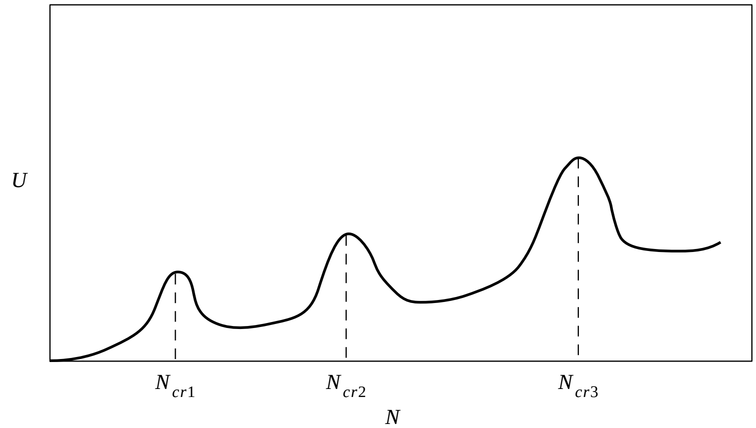

Figure 2. System strain energy versus rotating speed.

damped critical speeds allows one to determine the exact maximum unbalance responses, which need to be precisely predicted and controlled.

Most of the methods employed for calculating the damped critical speeds of rotor-bearing systems use the transfer matrix method or improved methods based on it [18–20]. For a complex system with branch structures or multi-level rotors however, the utilization of transfer matrices can become cumbersome as we have previously pointed out. In addition the complicated non-linear damping factor makes matters worse. To tackle this problem, a numerical algorithm is proposed based on a search for the maximum values of the system strain energy.

A typical energy curve comprising several critical speeds is illustrated in Figure 2. The idea of this method is based on one-dimensional search for the extrema of the multimodal function $U(n)$. Two main steps are proposed: (1) deciding upon each monomodal interval $[a_i, b_i]$ by one-dimensional searching procedure; (2) searching for the extreme in each interval, using one-dimensional optimization techniques.

This simple method allows one to obtain the damped critical speeds and the exact maximum values of the system strain energy. After comparing these extrema, one can determine the value of global maximum strain energy over a certain range of speeds, which can then be used as the optimal design objective.

2.4. CALCULATION OF TRANSMITTED FORCES

The use of rolling element bearings mounted on squeeze film dampers (SFDs) and centering springs like squirrel cages is the typical configuration used to attenuate unbalance responses. Centering springs can also be used to shift the critical speeds of the system. A typical support of this kind is illustrated schematically in Figure 3. Depending on the general type of structure occurring in aerospace engines and other machinery, we used short bearing theory to model the SFD, and assumed cavitation in the oil film (π film). The oil film's stiffness and damping can be represented by

$$k_0 = \frac{\mu R_d L^3 \Omega}{c^3} \cdot \frac{2\varepsilon}{(1-\varepsilon^2)^2}, \qquad d_0 = \frac{\mu R_d L^3}{c^3} \cdot \frac{\pi}{2(1-\varepsilon^2)^{3/2}}, \tag{13, 14}$$

where

$$\varepsilon = e/c. \tag{15}$$

The magnitude of the transmitted load is given by

$$F = (F_x^2 + F_y^2)^{1/2}, \tag{16}$$

where

$$F_x = k_e x - \Omega d_e y, \qquad F_y = \Omega d_e x + k_e y. \tag{17}$$

Here, k_e, d_e are the equivalent stiffness and damping of the support respectively, x, y are the displacements in two perpendicular directions at whirling plane, and Ω is the oil whirling speed. One can see from Figure 3 that k_e is determined by k_c, k_0 and bearing stiffness while d_e by d_c and d_0. In practical problems when bearing stiffness is relatively large, and $k_c \gg k_0$, $d_0 \gg d_c$, one can simply substitute k_e with k_c and d_e by d_0.

3. OPTIMIZATION MODEL AND METHODS

3.1. OBJECTIVE FUNCTION

Many factors need to be taken into account to achieve a successful design. Equations (7) to (11) show that the system strain energy reflects the distortion, strength (equation (8)) and deflection of the whole system, and the forces transmitted to the supports. It is therefore used as the objective function. Equations (9) and (12) show that the strain energy function is quadratic and positive definite, therefore possessing a unique minimum. This facilitates the optimization process. The system strain energy, either at a certain speed of operation or over a range of speeds, can be used as the objective function. Other design requirements can be treated as optimum constraints.

3.2. CONSTRAINTS

The constraints of optimization are defubed as follows.

(1) Constraints on the critical speeds n_{cr}: $n_{cr}^i \notin (n_l^j, n_u^j)$, where i is the order of critical speed; j is the number of restricted speed ranges; l and u stand for the lower and upper limit respectively.

(2) Constraints on the amplitudes and angular displacements of the disks: $p_d^i \leqslant p_{du}^i$ (i, the number of disks).

(3) Constraints on the shaft deflections: $y_s^i \leqslant y_{su}^i$ (i, the number of shafts).

(4) Constraints on the forces transmitted to the supports: $F^i \leqslant F_u^i$ (i, the number of supports).

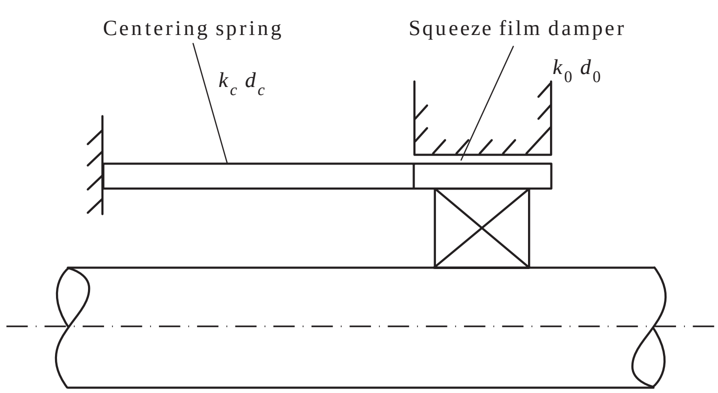

Figure 3. Layout of a typical rotor system support.

(5) The limits on support stiffnesses: $k_{el}^{i} \leqslant k_{e}^{i} \leqslant k_{eu}^{i}$ (i, the number of supports), where k_{el}^{i} is determined by the strength of the centering spring, and k_{eu}^{i} is usually the stiffness of the bearings.

(6) Constraints on dampers: this can be determined by the support's structure or the stability of the oil film. In this paper we use the constraints $0{\cdot}1\% \leqslant c_i/R_{di} \leqslant 0{\cdot}5\%$, $\varepsilon_i = e_i/c_i \leqslant 0{\cdot}4$ (i, the number of dampers).

(7) Constraints on system stability: system stability is considered via the damping coefficient of the system, $d_i > 0$ (i, the number of dampers).

This condition is, in fact, a sufficient one to ensure the stability in the optimization model we give.

3.3. VARIABLES

The equivalent stiffness k_e and equivalent damping coefficients d_e of the supports are used as variables. These two parameters have significant effects on the rotordynamic performance of the system. Besides, they are easy to adjust in practice. Suitable values of k_e can be obtained by changing the stiffness of the centering spring k_c, which depends on the structure of the spring, whilst d_e can be adjusted via the SFD damping coefficient d_0 which can be determined from the SFD's structural dimensions. Since the relationship between damping coefficients and unbalance responses must be correlated due to their non-linearity, the optimum value of the damping coefficients can hardly be used directly in actual practice. For the same reason, the limits on the coefficients are difficult to impose. To circumvent this problem, the following direct expression is employed, based on the SFD clearances:

$$c = \left(\left(\frac{\mu R_d L^3 \pi}{2 d_0}\right)^{2/3} + e^2\right)^{1/2}, \tag{18}$$

where R_d and L are respectively the radius and length of the bearings and treated as constants. Thus, once an optimal value d_0 is obtained, the amplitude e at the bearing point can be determined from the unbalance response calculations. Then c can be calculated using equation (18). The final optimal design parameters are the value of the centering spring stiffness k_c^* and the SFD's oil film clearance c^*.

Actually the effects of k_c and c on the system dynamic performance are interactive. A lower value of k_c will make the damper more efficient. However, when c is very small and the unbalance moment is large, a lower value of k_c amplifies the transmitted forces when the speed rises. The proposed approach allows one to obtain an optimal compromise.

3.4. MATHEMATICAL OPTIMIZATION MODEL AND METHODS

The mathematical optimization model is as follows:

$$\begin{aligned} &\text{minimize } f(X), \quad f(X) = U \\ &\text{subject to} \\ &g_i(X) \leqslant 0, \quad i = 1, \ldots, l, \\ &X \in S \subset R^n, \quad X = \{k_{e1}, \ldots, k_{en}, d_{e1}, \ldots, d_{em}\}, \end{aligned}$$

where l is the total number of constraints, and n, m are the number of equivalent stiffnesses and damping coefficients at the supports to be optimized, respectively. The method used in the optimization is the penalty function method combined with Powell's algorithm [21]. The one-dimensional optimization is carried out using the golden section search method.

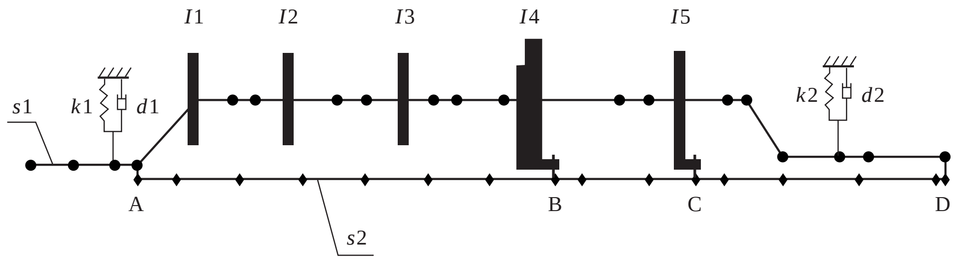

Figure 4. Simulation model of two-spool gas generator test equipment; k, stiffness; d, damper; I, inertia; s, subsystems; and ●, ◆, lumped mass nodes.

4. NUMERICAL EXAMPLES

4.1. TWO-SPOOL GAS GENERATOR TEST EQUIPMENT

To illustrate the proposed optimization strategy, an optimal design was performed on the rotor system of two-spool gas generator test equipment. The system consists of an outer gas generator rotor and a long inner shaft used as a center bolt. The outer rotor is rigidly fixed on the bolt at four joints by lock nuts. Both outer and inner shafts have various cross-sections and various stiffnesses. It has five disks on the generator and is supported by two bearings. Each bearing is supported by a centering spring incorporated with a squeeze film damper. The system has a total weight of 18·256 kg, a total length of 0·580 m with a bearing span of 0·461 m. The system was initially designed, following regular engineering routine, to cover two major working speed ranges: 18 000 ~ 20 000 r.p.m., 40 000 ~ 50 000 r.p.m. The initial design parameters and the system unbalance responses are tabulated in Tables 1–4. Previous tests and calculation results showed that the design was feasible. The second critical speed, however, was very close to the second working speed range, thus requiring further improvement.

The simplified simulation model is shown in Figure 4. The model is composed of two subsystems having a total of 39 nodes. They are rigidly connected at four points A, B, C and D. An unbalance of 5 gcm is assigned to disks 1, 4 and 5 with the same phase angles to simulate the most severe situation.

To optimize the system, two forbidden critical speeds ranges are imposed: 15 000 ~ 25 000 r.p.m. and 35 000 ~ 55 000 r.p.m. Furthermore, the design is expected to reduce simultaneously the vibration amplitude of the disks, the deflection of the shafts as well as the forces transmitted by the bearings to the supports. Two cases, based on minimum strain energy, are discussed below.

4.1.1. *Case* 1

Investigations were carried out to minimize the system strain energy over the whole speed range from 0 to 50 000 r.p.m. Such a design allows the rotor system to operate more smoothly during start-up and shut-down, and to pass safely through the critical speeds. Optimal and initial design parameters are compared in Tables 1 and 2, and include K_1, K_2, C_1/R_1, C_2/R_2 which are the stiffness and relative oil film clearances of the front and rear support, respectively. y_{imax} is the shaft deflection of two subsystems, P_{dmax} the maximum amplitude of disks, F_{imax} the maximum forces transmitted by the two bearings, and ΣF_{max} the total sum of the maximum bearing forces. Figure 5 illustrates the variations in the amplitudes of the two shafts along their total length.

TABLE 1

Comparison of support parameters and corresponding critical speeds: initial design versus optimal design (Case 1)

	$K_1 \times 10^7$ (N/M)	$K_2 \times 10^7$ (N/M)	C_1/R_{1d} %	C_2/R_{2d} %	N_{cr1} (r.p.m.)	N_{cr2} (r.p.m.)
Initial	1·25	1·97	0·300	0·300	10 865	38 150
Optimal	1·00	1·00	0·367	0·202	11 815	–

TABLE 2

Comparison of unbalance responses: initial design versus optimal design (Case 1)

	$U_{max} \times 10^{-3}$ (N · M)	$y_{1max} \times 10^{-3}$ (M)	$y_{2max} \times 10^{-3}$ (M)	$P_{dmax} \times 10^{-3}$ (M)	F_{1max} (N)	F_{2max} (N)	$\sum F_{max}$ (N)
Initial	17·241	0·0471	0·0460	0·0469	370·5	521·3	891·8
Optimal	6·951	0·0326	0·0315	0·0326	282·7	270·7	553·4
Reduction %	59·7	30·8	31·5	30·5	23·7	48·1	37·9

4.1.2. *Case* 2

The system is optimized at a working speed of 40 000 r.p.m. Such a design allows the system to have good rotordynamic performance over a stable, long-term working time.

The comparison between the optimal results and the initial design is shown in Tables 3 and 4, with the subscript "w" standing for the working speed. Figure 6 shows the corresponding variations in the whirling shaft deflections along their total length.

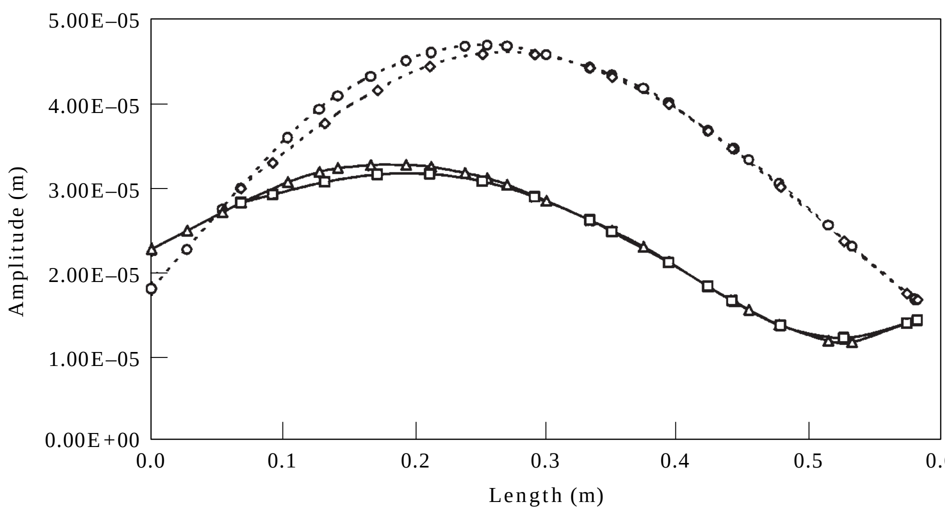

Figure 5. Amplitudes of the unbalance response of the initial and final optimal design in Case 1: —○—, initial $s1$; —◇—, initial $s2$; ——△——, optimal $s1$; ——□——, optimal $s2$.

TABLE 3

Comparison of support parameters and corresponding critical speeds: initial design versus optimal design (Case 2)

	$K_1 \times 10^7$ (N/M)	$K_2 \times 10^7$ (N/M)	C_1/R_1 %	C_2/R_2 %	N_{cr1} (r.p.m.)	N_{cr2} (r.p.m.)
Initial	1·25	1·97	0·300	0·300	10 865	38 150
Optimal	1·00	1·00	0·255	0·218	14 820	–

TABLE 4

Comparison of unbalance responses: initial design versus optimal design (Case 2)

	$U_w \times 10^{-3}$ (N · M)	$y_{1w} \times 10^{-3}$ (M)	$y_{2w} \times 10^{-3}$ (M)	$P_w \times 10^{-3}$ (M)	F_{1w} (N)	F_{2w} (N)	$\sum F_w$ (N)
Initial	7·242	0·0221	0·0198	0·0108	316·1	133·9	450·0
Optimal	5·345	0·0140	0·0187	0·0113	297·1	115·1	412·2
Reduction %	26·2	36·7	5·6	−4·6	6·0	14·0	8·4

4.2. DISCUSSION

Case 1 shows that, using the maximum strain energy of the system over a speed range as the objective function, the maximum deflections occurring in the two subsystems, as well as the maximum forces transmitted by each bearing, and the total transmitted forces, are all simultaneously reduced. Typical reductions vary from 23% up to 60% for the various parameters. Over most sections of the rotors, the whirling amplitude is also reduced (Figure 5). These observations seem to indicate that the system strain energy is an efficient quantity to use as the objective function. As a global parameter, it provides a general description of the severity of the working conditions of complex rotor-support systems.

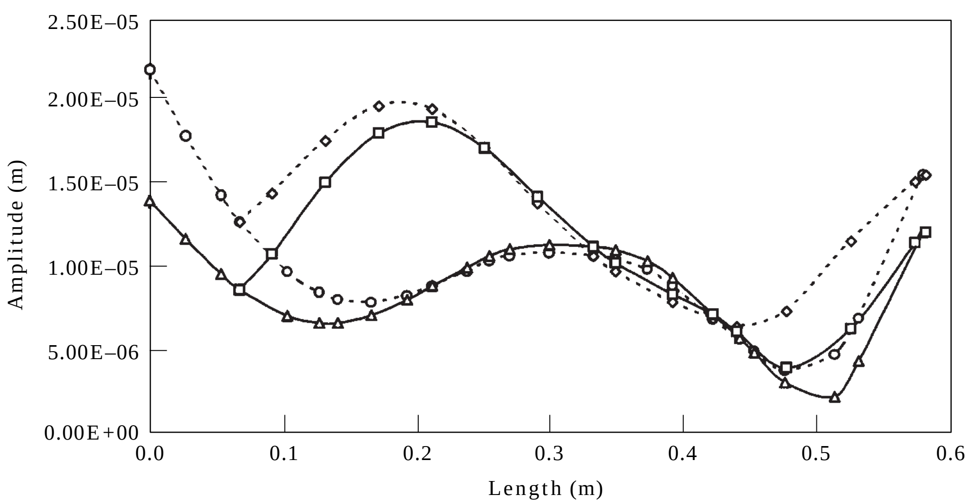

Figure 6. Amplitudes of the unbalance response of the initial and final optimal design in Case 2: ––○––, initial $s1$; ––◇––, initial $s2$; ——△——, optimal $s1$; ——□——, optimal $s2$.

When carrying out the system optimization at a fixed working speed, as in Case 2, one also finds a definite improvement for most of the previous design response parameters. However, the minimum system strain energy does not necessarily lead to a minimum amplitude at all locations or a minimum transmitted force for both bearings. In fact, from Table 4 and Figure 6, it can be noticed that slight increases in the amplitudes of some disks and shaft nodes may occur in the unbalance responses of the optimal design schema. Under some circumstances, these local parameters can be better controlled by adding appropriate local constraints. Nevertheless, the general distortion of the whole system is shown to be reduced in this specific application. Less distortion of the rotors ensures good stress conditions which tend to extend the structure's life. The advantages become apparent in flexible rotor systems with slender shafts whose strength is critical and where distortion is difficult to control. In this kind of situation, instead of using the strain energy of the whole system, one might choose the strain energy of rotors or shafts as the optimum objective to have a better control of these elements.

5. CONCLUSIONS

The optimization using system strain energy is shown to be a convenient way to handle the optimal design of complex flexible rotor-support systems. Compared with other objective functions, the use of the minimum system strain energy possesses the following advantages: (1) System strain energy is a global parameter describing the severity of the working conditions of complex rotor-support systems. (2) The distortion of the whole rotor system is minimized via the strain energy minimization. This feature is particularly attractive when dealing with flexible rotor systems where deflections are difficult to control and should prove helpful in extending the overall life of the structure. (3) A unique minimum can always be achieved because the strain energy function is both quadratic and positive definite.

Multiple constraints such as critical speeds, whirling amplitudes, transmitted forces and instability are necessary in order to achieve an optimal design of use in actual engineering practice. Support parameters such as the stiffness of the centering springs and clearances of SFDs are shown to be efficient variables to reach an optimal design in complex flexible systems. The developed strategy is believed to be useful during both the preliminary design and for modifications of existing machines.

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APPENDIX: NOMENCLATURE

c	squeeze film damper clearance
$[C]$	damping matrix
$[\bar{C}]$	damping matrix under modal co-ordinates
C_b	diagonal matrix of equivalent damping coefficients at boundary nodes
$[C_g]$	gyroscopic moment matrix
$[\bar{C}_g]$	gyroscopic moment matrix under modal co-ordinates
d_e	equivalent damping of support
d_c	damping of centering spring
d_0	equivalent damping of oil film
e	bearing eccentricity
F	transmitted force
I	unit matrix
$[K]$	stiffness matrix
$[\bar{K}]$	stiffness matrix under modal co-ordinates
k_0	equivalent stiffness of oil film
K_b	diagonal matrix of equivalent stiffness at boundary nodes
k_c	stiffness of centering spring
k_e	equivalent stiffness of support
K_s	static stiffness matrix of constrained subsystems at boundary nodes
L	damper length
$[M]$	mass matrix

$[\bar{M}]$	mass matrix under modal co-ordinates
$\{M_e\}$	unbalance moment
$\{p\}$	generalized displacement
R_d	radius of damper
U	system strain energy
y	lateral deflection
ω	rotational speed
ϕ	constrained undamped component mode shape
$[\Phi]$	assembly of component mode
δ	static deflection curve
Ω_c	subsystem undamped eigenvalue
σ	stress
ϵ	strain
ε	eccentricity ratio
μ	oil viscosity
Ω	whirling speed
*	complex conjugate transpose, optimal parameters